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Dear Friends

Please find herewith the analysis of the paragraphs 4.2.1.4 , 5.2.1.4 and 5.2.1.7 of the section 8 of the sporting code as I promise you during our last meeting of April 21.

A) Proposed redaction of the three paragraphs.

4.2.1.4 Distance traveled in outer space.

The outer space begins at the altitude 100 km and the distance is measured in the set of axis rotating with Earth along the ellipsoid at 100 km altitude between the point of entry in outer space and the point of exit (provided that the spacecraft remains above 100 km in the interval).

5.2.1.4 Unchanged

4.2.1.5 5.2.1.7. Distance traveled in outer space.

The outer space begins at the altitude 100 km and the distance is measured in the set of axis rotating with Earth along the ellipsoid at 100 km altitude between the point of entry in outer space and the point of exit (provided that the aerospacecraft remains above 100 km in the interval).

B ) Comments

The most natural definition of the points of entry and exit are their latitude, longitude and altitude in their ordinary meaning. However notice the following :

B 1 ) The mean sea level is only within 100 meters from a true ellipsoid.

B 2 ) The definition of latitudes and longitudes uses the vertical straight lines that are normal to the mean sea level and thus the approximation of the ellipsoid will gives small errors that will remain below 0.01% and below 400 meters.

I propose to consider these errors as negligible, and then a study with the approximation of the first order in the oblateness will be sufficient.

B 3 ) Theoretically, if the Earth was a true ellipsoid, the surface at 100km altitude would not exactly be an ellipsoid : there would be a slight elevation of 14cm at mid latitudes, but of course this elevation is completely negligible.

C ) Computation of the distance in outer space.

The limit between atmosphere and outer space will be considered as an oblate ellipsoid with the following dimensions, with a and b the Earth semi axes:

Semi major axis (in the equatorial plane) : $A = a + 100km = 6\ 478\ 137$ m

Semi minor and polar axis : $B = b + 100$ km $= 6\ 456\ 752.7$ m

The oblateness is then $\varepsilon = ( A - B ) / A = 1 / 302.933$

For an easy computation of the distance along this ellipsoid we need the corresponding "pseudo latitude" that is neither the geographic latitude nor the geocentric latitude nor even the parametric latitude but something in between that gives directly the distance along a meridian (with a relative error of the order of $\varepsilon^2$ ).

The geographic latitude, longitude and altitude of the point of entry in outer space will be $\varphi_1$ , $L_1$ and $h_1$ , with of course $h_1 = 100$ km.

The geographic latitude, longitude and altitude of the point of exit from outer space will be $\varphi_2$ , $L_2$ and $h_2$ , with $h_2 = 100$ km.

The geographic latitude $\varphi$ is related to the parametric latitude $\theta$ and the geocentric latitude $\lambda$ by the following :

$$\tan \varphi = ( a / b ) \tan \theta = ( a / b )^2 \tan \lambda \qquad (1)$$

Since ( a / b ) is very close to unity the three latitudes are very close to each other but the geographic latitude $\varphi$ gives the direction of the vertical (normal to the ellipsoid ) , the geocentric latitude $\lambda$ is the angle between the radius vector and the equatorial plane while the in between parametric latitude $\theta$ is easily related to the Cartesian coordinates :

A ) For the point $\varphi$ , L , 0 : $x = a \cos \theta \cos L$ ; $y = a \cos \theta \sin L$ ; $z = b \sin \theta$ (2)

B ) For the point $\varphi$ , L , h : $x = ( a \cos \theta + h \cos \varphi ) \cos L$

$$y = ( a \cos \theta + h \cos \varphi ) \sin L \quad ; \quad z = b \sin \theta + h \sin \varphi \qquad (3)$$

At the altitude h = 100 km we have also another parametric latitude $\Theta$ that (if we neglect the above 14 cm) is related to (3) by :

$( a \cos \theta + 100 \text{ km} . \cos \varphi ) = A \cos \Theta$ ; $z = b \sin \theta + 100 \text{ km} . \sin \varphi = B \sin \Theta$ (4)

Finally the “pseudo latitude” ,$\psi$ , is a little smaller than $\Theta$ , it is given by :

$$\tan \psi = ( B / A )^{1/2} \tan \Theta \qquad (5)$$

We don’t need this long succession of computations in order to obtain the pseudo latitude $\psi$ , it will be sufficient to compute the following that gives $\psi$ with an excellent accuracy :

$$\psi = \varphi - ( 0°.1421 ) . \sin 2\varphi = \varphi - ( 8' 32'' ) . \sin 2\varphi \qquad (6)$$

The distance D measured along the ellipsoid at 100 km altitude is then easily related to the latitudes and longitudes $\varphi_1$ , $L_1$ and $\varphi_2$ , $L_2$ and to their corresponding $\psi_1$ and $\psi_2$ :

A ) Compute the angle $\alpha$ between the points $\psi_1$ , $L_1$ and $\psi_2$ , $L_2$ in usual spherical coordinates :

$$\cos \alpha = \sin \psi_1 . \sin \psi_2 + \cos \psi_1 . \cos \psi_2 . \cos ( L_2 - L_1 ) \qquad 0° = \alpha = 180° \qquad (7)$$

This expression of $\alpha$ is equivalent to the following one that is sometimes more useful for small angles :

$$\sin^2 (\alpha / 2) = \sin^2 [(\psi_2 - \psi_1) / 2] + \sin^2 [(L_2 - L_1) / 2] . \cos\psi_1 . \cos\psi_2 \qquad (8)$$

We will also need the inclination i of the great circle of $\psi_1$ , $L_1$ and $\psi_2$ , $L_2$ on the equator :

$$\cos i = \cos\psi_1 . \cos\psi_2 . \left| \sin ( L_2 - L_1 ) \right| / \sin \alpha \qquad (9)$$

Independently of the angle i, the distance D is already given by the following excellent approximation ( with the angle $\alpha$ in radians) :

$$( A B )^{1/2} \alpha = D = A \alpha \qquad (10)$$

that is : $6\,467.435 \text{ km} \times \alpha = D = 6\,478 .137 \text{ km} \times \alpha$ (11)

If we need a better approximation (with relative errors of the order of $\varepsilon^2$ only) , we can use :

$$D = \alpha . \{ A - [ (A - B ) / 2 ] \sin^2 i \} \qquad (12)$$

that is : $D = \alpha . \{ 6\,472 .791 + 5 .346 \cos 2i \} \text{km}$ (13)

Notice that if $\alpha > 176°$ the approximation (11) remains valid, but the expression (13) looses its accuracy.

It is of course useless to write all these computations in an appendix of the sporting code and only the part after equation (5) is necessary. If you wish it I can write that appendix .

Final remark

The paragraph 3.9.4 of the sporting code is ambiguous : the set of axes is not defined and there is no “path of light” between the initial and final points of space-time. These question must be discussed at our next meeting.

Sincerely yours.

C. Marchal